e-content for students

B. Sc.(honours) Part 2paper 4

Subject:Mathematics

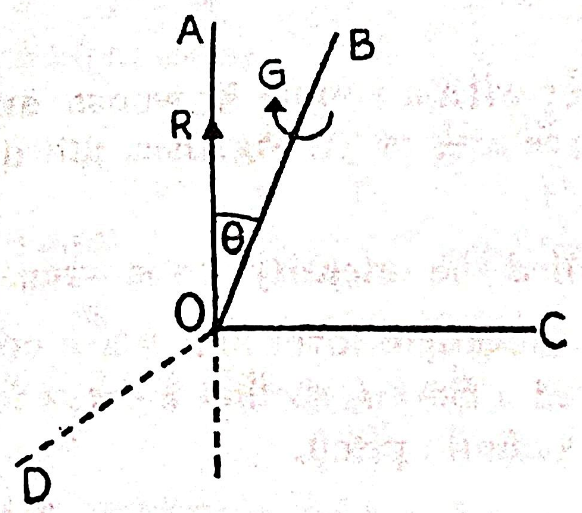
Topic:Poinsot's Central axis, Pitch

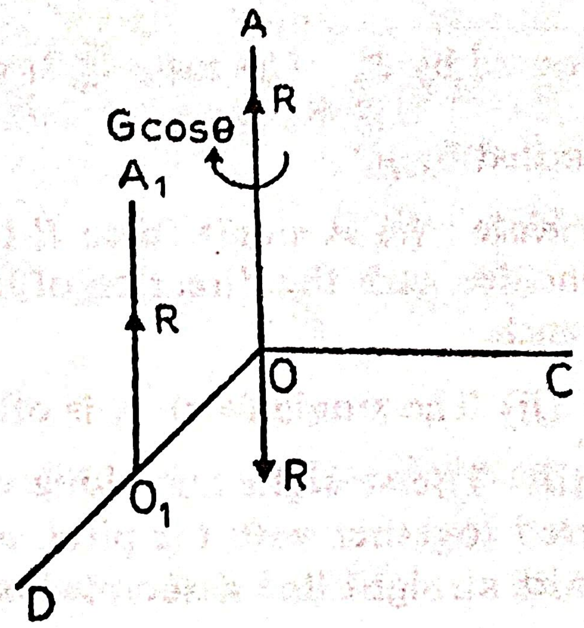
RRS college mokama

Poinsot's Central Axis, Pitch

Th **Show that any system of forces acting on a rigid body can be reduced to a single force together with a couple whose axis is along the direction of the force.**

Answer :

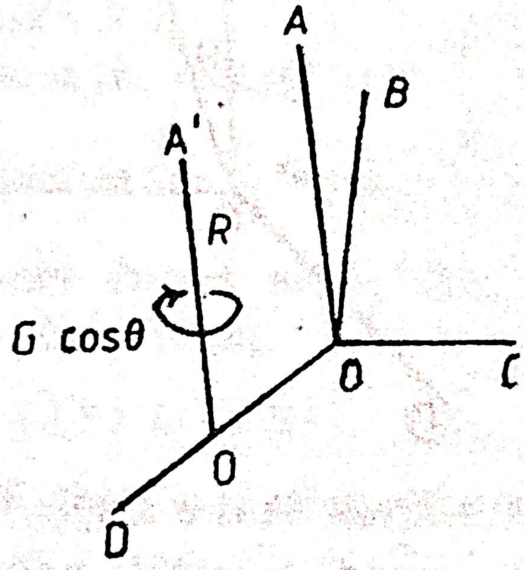




We know that any system of force acting at given points of a rigid body can be reduced to a single force R acting through an arbitrary chosen point O along OA, and a couple G whose axis OB passes through O. Let $\angle AOB = \theta$.

In the plane AOB draw OC perpendicular to OA, and draw OD perpendicular to the plane AOC.

Now the resolved part of G about OA as axis is $G\cos\theta$ and about OC as axis is $G\sin\theta$. Since the plane of the couple is normal to the axis of the couple, therefore, the latter couple $G\sin\theta$ about OC as axis acts in the plane AOD, and may therefore be replaced by any two equal unlike parallel forces of moment $G\sin\theta$.



Let one of these two forces be R acting at O in the direction opposite to OA. Then the other force must be equal to R acting parallel to OA at O_1 in OD, such that

$$R.OO_1 = G\sin\theta, \text{ i.e. } OO_1 = \frac{G\sin\theta}{R}.$$

The forces at O now balance. Hence we are left with one force R along O_1A_1 and a couple $G\cos\theta$ about OA as axis. Since O_1A_1 is parallel to OA, therefore the axis OA of the couple $G\cos\theta$ can be shifted to O_1A_1.

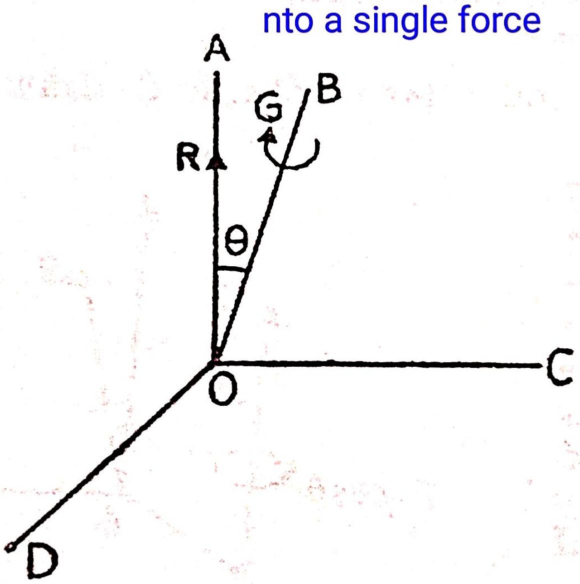
Hence the system reduces to a force R along O_1A_1 and a couple of moment $G\cos\theta$ about O_1A_1 as axis. Such a system is called a **Wrench** and the axis O_1A_1 is called **Poinsot's Central Axis.** $G\cos\theta$ is denoted by K. The ratio $\frac{K}{R}$ is called the **Pitch** of the wrench and is denoted by p.

Note : (i) A single force R together with a couple K whose axis coincides with the direction of the force are, taken together, called a **Wrench.**

(ii) The single force R is often called the **intensity** of the wrench.

(iii) The straight line along which the single force acts when considered together with the pitch is called a **Screw,** so that a **screw** is a definite straight line associated with a definite pitch.

Th:Find the condition that a given system of force should compound into a single force



Answer : We know that any system of forces acting at given points of a rigid body can be reduced to a single force R acting through an arbitrary chosen point O along OA, and a couple G whose axis OB passes through O. Let $\angle AOB = \theta$.

Now the resolved part of R along OB is $R\cos\theta$ and perpendicular to OB is $R\sin\theta$.

We know that a single force and a couple acting in the same plane upon a rigid body cannot produce equilibrium but are equivalent to the single force acting in a direction parallel to its original direction.

Hence $R\sin\theta$ and the parallel forces of G are equivalent to a parallel force $R\sin\theta$ which does not pass through O, and therefore cannot, in general, compound with $R\cos\theta$ into a single force, because only concurrent forces can be compounded into a single force through that point.

But if $R\cos\theta=0$, i.e. if $\cos\theta=0$, i.e. if $\theta=\frac{\pi}{2}$, then we are left with a single force $R\sin\theta$ which does not pass through O. Therefore the angle between OA, whose direction cosines are $\frac{X}{R}, \frac{Y}{R}, \frac{Z}{R}$, and OB, whose direction cosines are $\frac{L}{G}, \frac{M}{G}, \frac{N}{G}$, is a right angle.

$$\therefore \quad \frac{X}{R}\cdot\frac{L}{G}+\frac{Y}{R}\cdot\frac{M}{G}+\frac{Z}{R}\cdot\frac{N}{G}=0 \Rightarrow XL+YM+ZN=0.$$

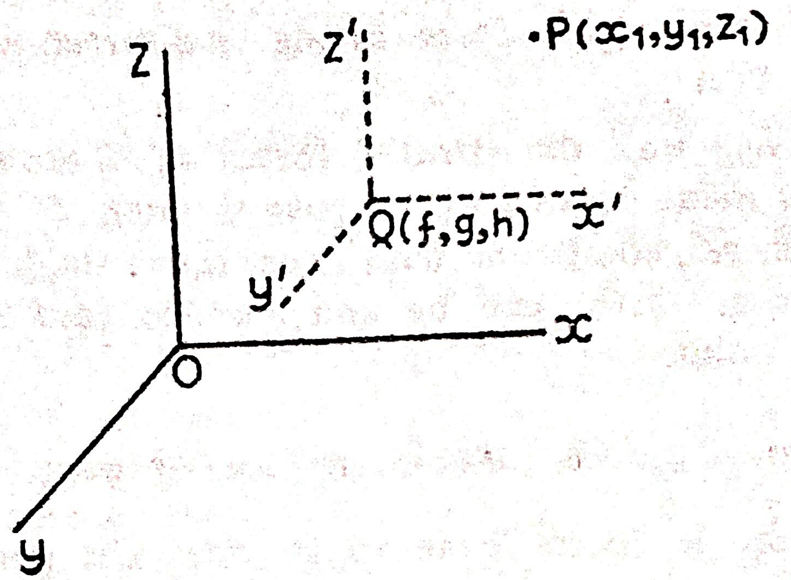
which is the required condition.

Th; Find the equation of the Central Axis of any given system of force

Answer : Let X, Y, Z be the component forces along Ox, Oy, Oz and L, M, N the component couples about them.

Let $\qquad R^2 = X^2 + Y^2 + Z^2$ and $G^2 = L^2 + M^2 + N^2$.

When the axis of the couple G and the line of action of R coincide, the axis is called the central axis.



Let Q be a point on the central axis, whose co-ordinates referred to Ox, Oy, Oz as axes be (f, g, h).

Let P be a point whose co-ordinates are (x_1, y_1, z_1) with respect to Ox, Oy, Oz and hence (x_1-f, y_1-g, z_1-h) with respect to Qx', Qy', Qz', which are parallel to Ox, Oy, Oz respectively.

Now, the moment of the couple about Qx'

$$=\Sigma[(y_1-g)Z_1-(z_1-h)Y_1]=\Sigma(y_1Z_1-z_1Y_1)-g\Sigma Z_1+h\Sigma Y_1$$
$$=L-gZ+hY=L' \text{ (say)}.$$

Similarly the moment of the couple about Qy'

$$=M-hX+fZ=M' \text{ (say)},$$

and the moment of the couple about Qz'

$$=N-fY+gX=N' \text{ (say)}.$$

Also the components of the resultant force are the same for all points such as Q, and are thus X, Y, and Z.

Since Q lies on the central axis, therefore the direction cosines of the axis of the couple corresponding to it are proportional to those of the resultant force.

$$\therefore \quad \frac{L'}{X}=\frac{M'}{Y}=\frac{N'}{Z}$$

$$\Rightarrow \quad \frac{L-gZ+hY}{X}=\frac{M-hX+fZ}{Y}=\frac{N-fY+gX}{Z}$$

$$=\frac{LX+MY+NZ}{X^2+Y^2+Z^2}=\frac{R.K}{R^2}=\frac{K}{R}.$$

Hence the equation of the locus of the point (f, g, h), i.e. the required equation of the central axis is

$$\frac{L-yZ+zY}{X}=\frac{M-zX+xZ}{Y}=\frac{N-xY+yX}{Z}$$

$$=\frac{K}{R}=\text{the pitch } p \text{ of the wrench.}$$